On propagation through long step tapers

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1. Abstract

In integrated optics, tapered waveguides usually possess step changes in waveguide width or other dimensions, due to lithographic limitations. We present a theoretical asymptotic analysis of the effect of these steps when compared with the ideal smooth taper. We present some figures for the size of the effect, conclude that the effect is significant in certain situation and that making the taper too long will reduce its performance, or in the worst instance cause the taper to function as badly as a butt join.

2. Introduction.

Tapered waveguides are commonly used to channel a guided light signal from an input waveguide into an output waveguide with different cross section, for example to increase the efficiency of the coupling between a laser chip and an optical fibre (so called "spot size convertors") [1,2]. Tapers are usually fabricated by a lithographic process. This is particularly true when making tapered structures in integrated optical components. In practice, due to the finite resolution of the lithographic mask, the resulting tapers will contain small uniform steps in the profile. The presence of these small steps will alter the transmission properties of the taper. This paper investigates the effect of these small steps and demonstrates that their effect can have

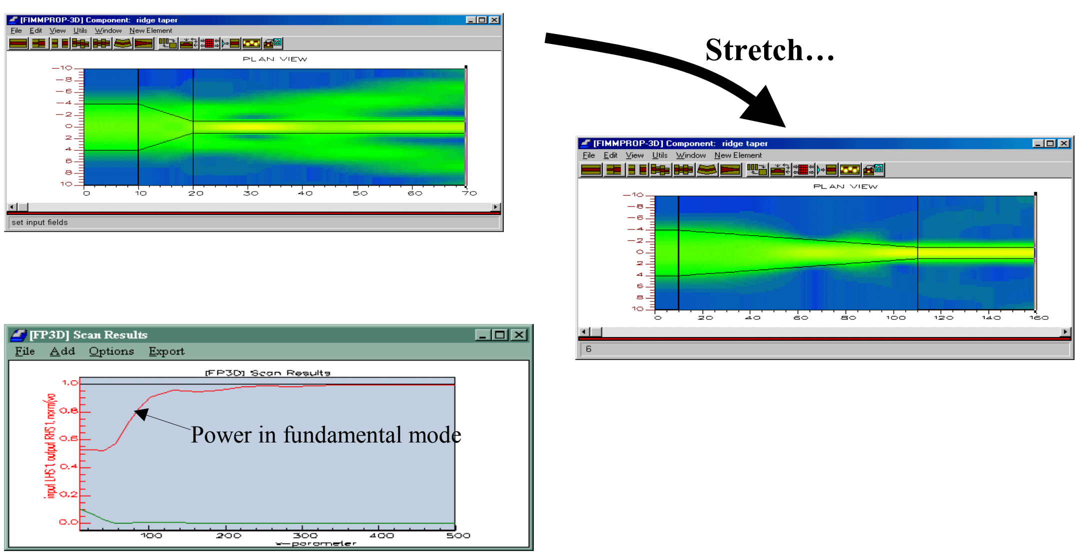

Figure 1. As a smooth taper is stretched power will be conserved in injected mode, as is seen from this graph of power remaining in the fundamental mode vs. taper length.

a very significant effect, even for steps of 1/20th the operating wavelength (0.05μm at the 1μm to 1.5μm band) – generally the limit of conventional technology.

It is well known that *smooth tapers*, i.e. waveguides whose cross section changes smoothly from the input to the output waveguide, become adiabatic as their length tends to infinity. Adiabatic in this context means that the power in each local guided mode is conserved along the taper. Efficient transmission into the output waveguide is therefore achieved by constructing a long enough taper. This behaviour of *smooth tapers* can be shown using coupled mode theory: the cross coupling between non-degenerate local modes tends to zero as the taper length increases. Hence if the fundamental mode of the input waveguide is excited, then the shape of this mode will deform continuously along the taper into the fundamental mode of the output waveguide without any power being transferred to other modes (figure 1).

In a *step taper* lateral dimensions vary in a step-like fashion due to the manufacturing process. In such a taper, it is observed that as the length of the taper increases the modal cross coupling decreases, as it does in a *smooth taper*, but then proceeds to oscillate without ever converging to zero (figure 3). A-priori knowledge of the behaviour of the power transmission for varying taper length therefore becomes important when designing a *step taper* with a specified minimum transmission efficiency.

In what follows we present a study of light propagating through *step tapers*. We use asymptotic analysis to derive an approximate solution to the modal evolution equations valid for a large number of steps. We show that *step tapers* exhibit rather different behaviour from their smooth counterparts as their length is increased – the finite step size will cause resonant peaks at specific taper lengths in the coupling between forward and backward guided modes. The amplitude of the peaks will be the same when the taper has zero length (the butt join case). The resonant taper lengths at which these peaks occur depend on the value of the guided mode propagation constants and increase with the number of steps, so that the resonant points disappear in the limit of the *step taper* becomes smooth. The analytic results obtained provide practical design criteria for building *step tapers* with required transmission characteristics.

3. *The step taper*

The following analysis is valid for *step tapers* of arbitrary cross section. We consider a *step taper*, which could be a ridge or buried waveguide in a chip surrounded by a cladding or a base layer. Both the core and the cladding are assumed to have a sufficiently high refractive index with respect to the surrounding medium so that it will have a set of local guiding modes at any point along the "z" axis (figure 2). Numerical experiments (figure 3) show that as the taper length is increased, oscillations appear in the transmitted power. By looking in detail at the propagating field in tapers with lengths giving low and high loss respectively (figure 4), we can clearly see the resonant nature of the mode coupling. For the low loss taper, the power begins to couple out of the excited mode, but couples back in before the end is reached. For the high loss taper, the power couples out of the excited mode at a constant rate.

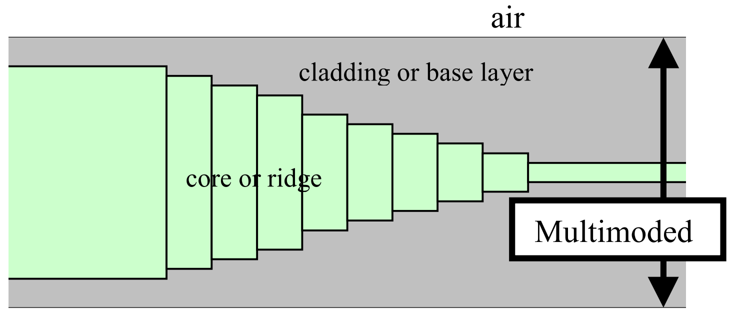

Figure 2. The entire structure is assumed to be a multimoded region at all "z" positions.

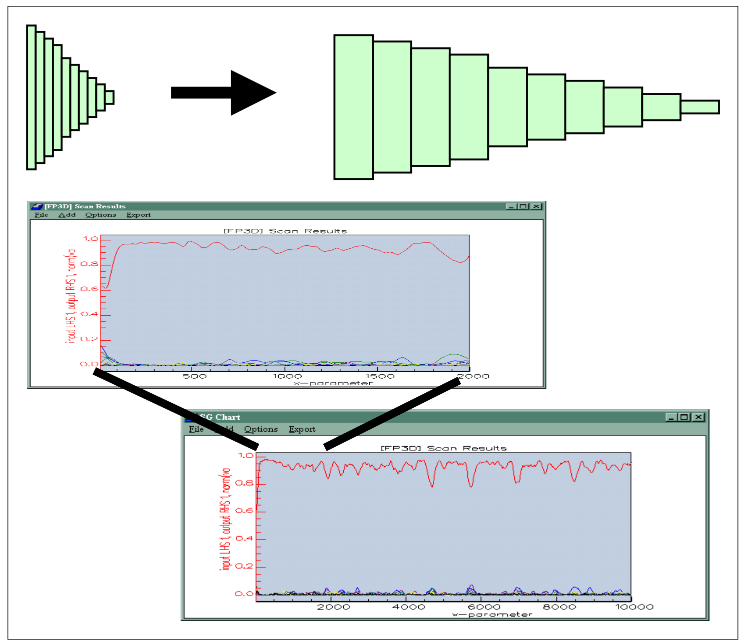

Figure 3. Graph of the power transmission into the output fundamental mode as the step taper is stretched.

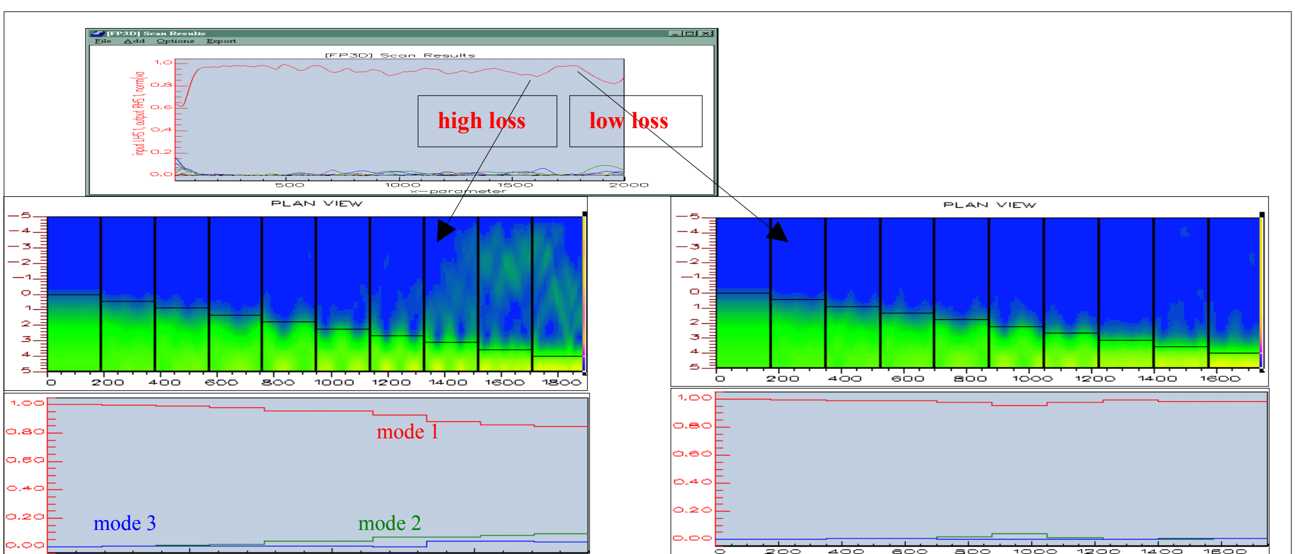

Figure 4. plot of fields and mode power for taper lengths that are resonant (maximum power coupling from fundamental into higher order modes) and non resonant (minimum coupling). Notice how in the latter case the transmitted power swings back into mode 1.

4. Solution using local modes.

In each uniform cross section of the *step taper* the field can be expressed in terms of its modal expansion.

$$\text{(1)}\qquad \begin{aligned} \mathbf{E}(x,y,z) &= \sum_{k=1}^{J}\left(a_k e^{i\beta_k z} + b_k e^{-i\beta_k z}\right)\mathbf{E}_k(x,y) + \mathbf{E}_{rad}(x,y,z) \\ \mathbf{H}(x,y,z) &= \sum_{k=1}^{J}\left(a_k e^{i\beta_k z} - b_k e^{-i\beta_k z}\right)\mathbf{H}_k(x,y) + \mathbf{H}_{rad}(x,y,z) \end{aligned}$$

where

$[\mathbf{E}_k, \mathbf{H}_k]$ are the discrete set of the J guided and evanescent **eigenmodes,**

$\beta_k(z)$ their respective **propagation constants,**

$\mathbf{E}_{rad}, \mathbf{H}_{rad}$ the radiation components.

Continuity of the tangential fields across consecutive sections m, m+1 and the orthogonality of the modes implies that

$$\text{(2)}\qquad \begin{cases} \mathbf{a}^{m+1} + \mathbf{b}^{m+1} = \mathbf{O}^{m+1,m}\left(e^{i\boldsymbol{\beta}^m l}\mathbf{a}^m + e^{-i\boldsymbol{\beta}^m l}\mathbf{b}^m\right) + \mathbf{e}_{rad}^{m+1,m} \\ \mathbf{a}^{m+1} - \mathbf{b}^{m+1} = \mathbf{O}^{m,m+1^T}\left(e^{i\boldsymbol{\beta}^m l}\mathbf{a}^m - e^{-i\boldsymbol{\beta}^m l}\mathbf{b}^m\right) + \mathbf{h}_{rad}^{m+1,m} \end{cases};$$

where:

- $\boldsymbol{\beta}^m = diag\left[\beta_1^m, \beta_2^m, \ldots, \beta_J^m\right]$,
- $\mathbf{a}^m, \mathbf{b}^m$ are the coefficients of the forward and backward modes at the beginning of step "m",
- "l" is the step length,
- $\mathbf{O}^{m+1,m}$ is the cross coupling matrix at the interface between steps given by $O_{jk}^{m+1,m} = \int \mathbf{E}_k^{m+1} \wedge \mathbf{H}_j^m ds$
- $\mathbf{e}_{rad}^{m+1,m}, \mathbf{h}_{rad}^{m+1,m}$ are the radiation contributions given by

$$e_{rad}^{m+1,m}{}_j = \int \left(\mathbf{E}_{rad}^m - \mathbf{E}_{rad}^{m+1}\right) \wedge \mathbf{H}_j^m ds \quad ; \quad h_{rad}^{m+1,m}{}_j = \int \left(\mathbf{H}_{rad}^m - \mathbf{H}_{rad}^{m+1}\right) \wedge \mathbf{E}_j^m ds$$

By rearranging (2) to make $\mathbf{a}^{m+1}, \mathbf{b}^{m+1}$ explicit we obtain

$$\text{(3)}\qquad \begin{cases} \mathbf{a}^{m+1} = \mathbf{S}^{m+1,m} e^{i\boldsymbol{\beta}^m l}\mathbf{a}^m + \mathbf{A}^{m+1,m} e^{-i\boldsymbol{\beta}^m l}\mathbf{b}^m + \frac{1}{2}\left(\mathbf{e}_{rad}^{m+1,m} + \mathbf{h}_{rad}^{m+1,m}\right) \\ \mathbf{b}^{m+1} = \mathbf{S}^{m+1,m} e^{-i\boldsymbol{\beta}^m l}\mathbf{b}^m + \mathbf{A}^{m+1,m} e^{i\boldsymbol{\beta}^m l}\mathbf{a}^m + \frac{1}{2}\left(\mathbf{e}_{rad}^{m+1,m} - \mathbf{h}_{rad}^{m+1,m}\right) \end{cases}$$

with

$$\text{(4)}\qquad \mathbf{S}^{m+1,m} = \frac{1}{2}\left(\mathbf{O}^{m+1,m} + \mathbf{O}^{m,m+1^T}\right),\ \mathbf{A}^{m+1,m} = \frac{1}{2}\left(\mathbf{O}^{m+1,m} - \mathbf{O}^{m,m+1^T}\right)$$

- the symmetric and anti symmetric coupling matrices.

5. Asymptotic analysis.

The aim is to derive, given an input $\mathbf{a}^1$ at the beginning of the *step taper*, an expression for $\mathbf{a}^m, \mathbf{b}^m$ and the excitation of the radiation components. In principle this is done by combining system (3) at each interface. Unfortunately, the presence of the radiation components prevents us from doing this. We therefore assume from the outset that coupling into radiation modes can be ignored (we will see later when this is justified, and what happens if the coupling is significant). In this case system (3) reduces to:

$$(5) \qquad \begin{cases} \mathbf{a}^{m+1} = \mathbf{S}^{m+1,m} e^{i\boldsymbol{\beta}^m l}\mathbf{a}^m + \mathbf{A}^{m+1,m} e^{-i\boldsymbol{\beta}^m l}\mathbf{b}^m \\ \mathbf{b}^{m+1} = \mathbf{S}^{m+1,m} e^{-i\boldsymbol{\beta}^m l}\mathbf{b}^m + \mathbf{A}^{m+1,m} e^{i\boldsymbol{\beta}^m l}\mathbf{a}^m \end{cases}$$

Now, let M be the number of taper steps. If we assume many steps (large M), then the difference between successive steps is small. The cross coupling is therefore also small so that the coupling matrices (4) can be separated into quantities of different scale:

$$(6) \qquad \mathbf{S}^{m+1,m} = \mathbf{S}_d^{m+1,m} + \mathbf{S}_o^{m+1,m}$$

where the diagonal part $\mathbf{S}_d^{m+1,m} \sim \mathbf{I} + O(M^{-2})$ and the off diagonal part $\mathbf{S}_o^{m+1,m} \sim O(M^{-1})$, while $\mathbf{A}^{m+1,m} \sim O(M^{-1})$.

This suggests looking for solution to $\mathbf{a}^m, \mathbf{b}^m$ in the form

$$(7) \qquad \mathbf{a}^m = \mathbf{a}_0^m + \mathbf{a}_1^m + \cdots \quad ; \quad \mathbf{b}^m = \mathbf{b}_0^m + \mathbf{b}_1^m + \cdots$$

where we assume a priori that $\mathbf{a}_k^m, \mathbf{b}_k^m \sim O(M^{-k})$ in all slices m. This will be justified once the asymptotic solution is found. Under this assumption we can equate terms of the same order.

order 0: $\begin{cases} \mathbf{a}_0^{m+1} = e^{i\boldsymbol{\beta}^m l}\mathbf{a}_0^m \\ \mathbf{b}_0^{m+1} = e^{-i\boldsymbol{\beta}^m l}\mathbf{b}_0^m \end{cases}$ which gives

$$(8) \qquad \mathbf{a}_0^m = e^{i\sum_1^{m-1}\boldsymbol{\beta}^r l}\mathbf{a}_0^1 \quad ; \quad \mathbf{b}_0^m = e^{-i\sum_1^{m-1}\boldsymbol{\beta}^r l}\mathbf{b}_0^1$$

At this point we assume that there is no input field from the rhs ($\mathbf{b}_0^M = 0$), so that from (8), $\mathbf{b}_0^m = 0$ for all m. Hence:

order 1: $\begin{cases} \mathbf{a}_1^{m+1} = e^{i\boldsymbol{\beta}^m l}\mathbf{a}_1^m + \mathbf{S}_o^{m+1,m} e^{i\boldsymbol{\beta}^m l}\mathbf{a}_0^m \\ \mathbf{b}_1^{m+1} = e^{-i\boldsymbol{\beta}^m l}\mathbf{b}_1^m + \mathbf{A}^{m+1,m} e^{i\boldsymbol{\beta}^m l}\mathbf{a}_0^m \end{cases}$

the top line can be rewritten as

$$(9) \qquad \begin{cases} \boldsymbol{\gamma}_1^{m+1} = \boldsymbol{\gamma}_1^m + e^{-i\sum_1^m\boldsymbol{\beta}^r l}\mathbf{S}_o^{m+1,m} e^{i\sum_1^m\boldsymbol{\beta}^r l}\mathbf{a}_0^1 \\ \boldsymbol{\mu}_1^{m+1} = \boldsymbol{\mu}_1^m + e^{i\sum_1^m\boldsymbol{\beta}^r l}\mathbf{A}^{m+1,m} e^{i\sum_1^m\boldsymbol{\beta}^r l}\mathbf{a}_0^1 \end{cases} \quad \text{with} \quad \mathbf{a}_1^m = e^{i\sum_1^{m-1}\boldsymbol{\beta}^r l}\boldsymbol{\gamma}_1^m \,;\; \mathbf{b}_1^m = e^{-i\sum_1^{m-1}\boldsymbol{\beta}^r l}\boldsymbol{\mu}_1^m$$

Note that (8) is the solution for a straight uniform waveguide, while (9) provides the O(1/M) correction. In order to solve the latter explicitly we need to make a couple of extra assumptions, namely that the propagation constants and the cross coupling matrices are approximately constant from one section to the next, so that we can write:

$$\boldsymbol{\beta}^m \approx \overline{\boldsymbol{\beta}} \qquad (= \text{some average value})$$

and

$$\mathbf{S}_o^{m+1,m} \approx \frac{\mathbf{K}_F}{M} \quad ; \quad \mathbf{A}^{m+1,m} \approx \frac{\mathbf{K}_B}{M}$$

where $\mathbf{K}_F, \mathbf{K}_B$ are the cross-coupling matrices for the equivalent butt for the forward and backward propagating modes respectively.

So from (9):

$$\boldsymbol{\gamma}_1^M = e^{-i\bar{\boldsymbol{\beta}}l} \frac{\mathbf{K}_F}{M} e^{i\bar{\boldsymbol{\beta}}l} \mathbf{a}_0^1 + \cdots + e^{-i\bar{\boldsymbol{\beta}}(M-1)l} \frac{\mathbf{K}_F}{M} e^{i\bar{\boldsymbol{\beta}}(M-1)l} \mathbf{a}_0^1$$

$$\boldsymbol{\mu}_1^M = e^{i\bar{\boldsymbol{\beta}}l} \frac{\mathbf{K}_B}{M} e^{i\bar{\boldsymbol{\beta}}l} \mathbf{a}_0^1 + \cdots + e^{i\bar{\boldsymbol{\beta}}(M-1)l} \frac{\mathbf{K}_B}{M} e^{i\bar{\boldsymbol{\beta}}(M-1)l} \mathbf{a}_0^1$$

Now suppose that the input field to the taper is the excited fundamental mode of the input waveguide, i.e. $\mathbf{a}_0^1 = [1,0,0,...]$. Then we obtain

$$\boldsymbol{\gamma}_{1\ 1}^M = 0 \qquad \text{(as } K_{F11} = 0 \text{ by definition of } \mathbf{S}_o^{m+1,m}\text{)},$$

$$\begin{cases} \boldsymbol{\gamma}_{1\ j}^M = \dfrac{K_{Fj1}}{M}\left[e^{i(\bar{\beta}_1-\bar{\beta}_j)l} + \cdots + e^{i(\bar{\beta}_1-\bar{\beta}_j)(M-1)l}\right] & (j>1) \\ \mu_{1\ j}^M = \dfrac{K_{Bj1}}{M}\left[e^{i(\bar{\beta}_1+\bar{\beta}_j)l} + \cdots + e^{i(\bar{\beta}_1+\bar{\beta}_j)(M-1)l}\right] & (j \ge 1) \end{cases}$$

The latter can be summed to give:

$$(10) \qquad \begin{cases} \boldsymbol{\gamma}_{1\ j}^M = \dfrac{K_{Fj1}}{M} e^{i(\bar{\beta}_1-\bar{\beta}_j)l}\left[\dfrac{1-e^{i(\bar{\beta}_1-\bar{\beta}_j)l(M-2)}}{1-e^{i(\bar{\beta}_1-\bar{\beta}_j)l}}\right] & (j>1) \\ \mu_{1\ j}^M = \dfrac{K_{Bj1}}{M} e^{i(\bar{\beta}_1+\bar{\beta}_j)l}\left[\dfrac{1-e^{i(\bar{\beta}_1+\bar{\beta}_j)l(M-2)}}{1-e^{i(\bar{\beta}_1+\bar{\beta}_j)l}}\right] & (j \ge 1) \end{cases}$$

Hence total power lost from mode 1 in a *step taper* of length L with M steps is obtained by summing up the power coupling into all forward and backward propagating modes:

$$(11) \qquad P_{loss} = \left|\mathbf{a}_1^M\right|^2 + \left|\mathbf{b}_1^M\right|^2 = \left|\boldsymbol{\gamma}_1^M\right|^2 + \left|\boldsymbol{\mu}_1^M\right|^2 = \sum_{j=-J, j\neq 0}^{J} \left[\frac{K_{j1}}{M} \frac{\sin\dfrac{(\bar{\beta}_1-\bar{\beta}_j)L}{2}}{\sin\dfrac{(\bar{\beta}_1-\bar{\beta}_j)L}{2M}}\right]^2$$

where we have replaced the step length "*l*" with "*L/M*" and have combined forward and backward propagating components by replacing:

- K_{Fj1}, K_{Bj1} with K_{j1}, K_{-j1} (j>0) respectively,
- $-\bar{\beta}_j$ with $\bar{\beta}_{-j}$ (j>0)

A few points to note:

1. The above expression is only valid if the individual terms in (10) remain of order *1/M*. This is true in principle only so long as the denominator in each term (10), and therefore (11), remains bounded away from zero.

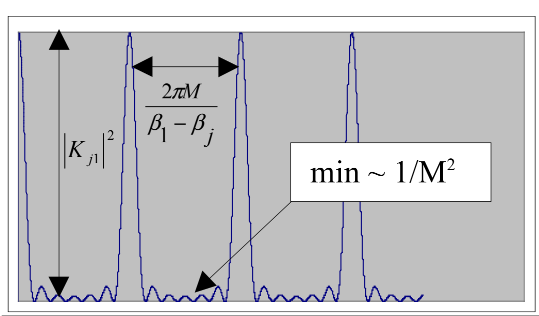

Figure 5. plot of j'th component in (11)

2. At these singular points (figure 5) the power loss blows up. The peak power at the singular point for the j'th term in (11) occurs at lengths

(12) $$L = \frac{2\pi M}{\bar{\beta}_1 - \bar{\beta}_j} n \qquad (n=0,1,2,\ldots)$$

3. On performing a local expansion around these singular lengths we find that the amplitude of the j'th term is exactly $\left|K_{j1}\right|^2$. This is the same as the cross coupling for the butt join between mode 1 and j (forward mode if j>1, backward if j<0). Strictly speaking these values violate our initial assumption that the perturbed values remain of order "1/M". Nevertheless they do provide a good idea of the behaviour of these solution.
4. The power loss (11) is a superposition of the individual modal interactions. For a zero length taper all interactions are at their singular peaks, so that the power loss at L=0 is $P_{loss}\big|_{L=0} = \sum_{j=-J,j\neq 0}^{J}\left|K_{j1}\right|^2$, which is exactly the power loss expected in a butt join. This confirms the validity of this expression even when the asymptotic conditions are not met.
5. As the length is gradually increased the power loss decreases exactly as it would do for the *smooth taper*, but will then rises again once the length reaches the first resonant length L_0, which corresponds to the fundamental BACKWARD travelling mode. This is given by (12) with n=1,j=-1:

(13) $$L_0 = \frac{\pi M}{\bar{\beta}_1}$$

   and the power coupled into this backward travelling mode is of the order $\left|K_{R11}\right|^2$ - the coupling coefficient from forward mode 1 to backward mode 1 for the equivalent butt join.
6. This first resonant length L_0 will occur even if there is only one guided mode present (J=1 in (11)). If the *step taper* structure in figure 2 is indeed multimodal (J>1) then as the *step taper* is stretched peaks will occur at the subsequent resonant lengths (12).
7. The resonant interactions with the backward propagating modes will be significant if the reflection coefficients $\mathbf{K}_B$ of the equivalent butt join are significant. This will typically occur when the dimensions of the end cross sections are very different, and/or in the presence of a large refractive index contrast between the core and cladding. See figure 8 for such an example.

8. As we increase the number of steps the resonant lengths increase (figure 6), so that in the

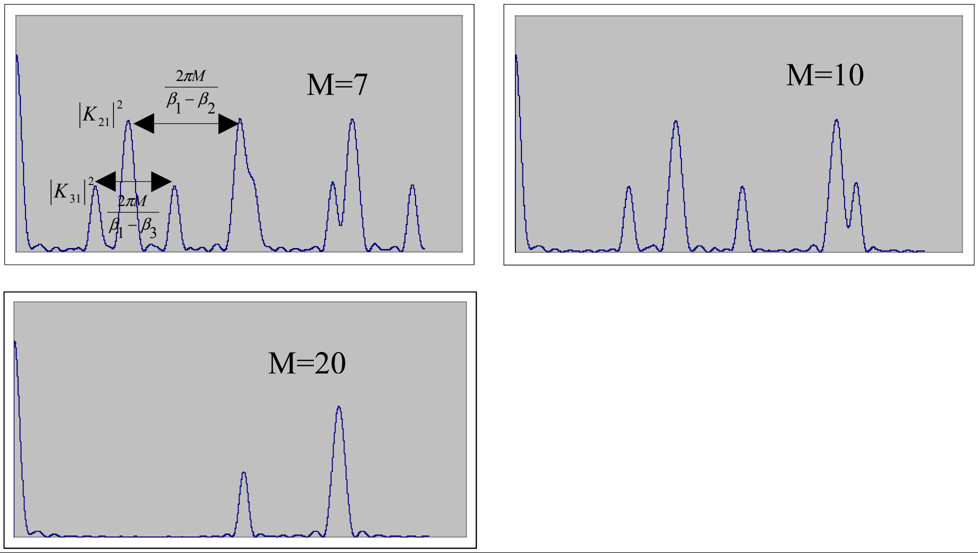

Figure 6. Power loss (11) vs. taper length L taking into account the first three modes for different number of steps M.

limiting smooth case (infinite M) the only resonant peak remaining is the one at zero length (the butt join). The power loss in this case is given by setting M to infinity in (11):

$$P_{loss}\Big|_{M\to\infty} = \sum_{j=-J,j\neq 0}^{J}\left[\frac{2K_{j1}}{(\overline{\beta}_1-\overline{\beta}_j)L}\sin\frac{(\overline{\beta}_1-\overline{\beta}_j)L}{2}\right]^2 \tag{14}$$

This is precisely the expression derived using the continuous coupled mode equations found in various texts, such as [3], except that this result is more general as it includes interactions among all forward and backward modes instead of just two forward modes found using the continuous approach.

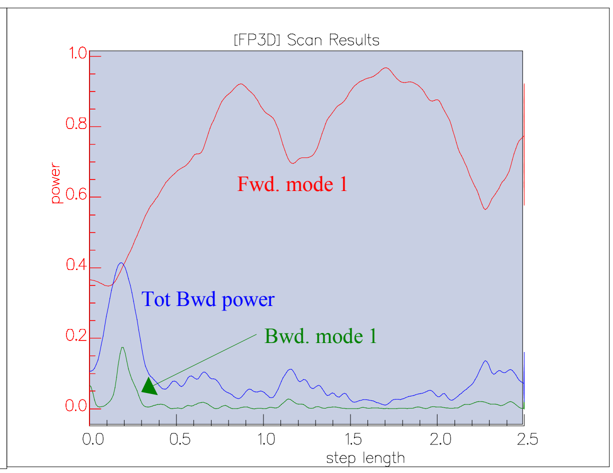

Figure 8. Length scan of a step taper showing power remaining in fundamental forward and backward mode, and total reflected guided power. The step taper has 8 steps. The taper core has average index of 3 and cladding index of 1.5. The input and output core sizes are 1.5µm, 3µm respectively. Wavelength is 1.55µm. These are realistic values for silica rib waveguides. The high contrast in dimensions and refractive index cause peaks (e.g. step length=1.15) of the order of the predicted value – i.e. the butt join coupling (at step length=0). Note the peak for the very small step 0.2µm, which is even higher than this predicted value.

6. Radiation modes.

The results obtained up to now ignore the interaction with radiation modes. This is justified if the refractive index jump at cladding/air interface in the structure in consideration (figure 2) is

sufficiently high to prevent any power escaping from the cladding. This will be typically the case given the materials used for fabricating ridge waveguides.
However if the radiation is significant, it can always be modelled by enclosing the entire device by reflecting side boundary conditions. If the side boundaries are at a finite distance from the device, the radiation modes will be discretised, and can therefore be included in expression (11) as a second sum:

$$(15)\qquad P_{loss} = \sum_{j=-J,j\neq 0}^{J}\left[\frac{K_{j1}}{M}\frac{\sin\dfrac{(\overline{\beta}_1-\overline{\beta}_j)L}{2}}{\sin\dfrac{(\overline{\beta}_1-\overline{\beta}_j)L}{2M}}\right]^2 + \sum_{j<-J,j>J}\left[\frac{K_{j1}}{M}\frac{\sin\dfrac{(\overline{\beta}_1-\overline{\beta}_j)L}{2}}{\sin\dfrac{(\overline{\beta}_1-\overline{\beta}_j)L}{2M}}\right]^2$$

the second sum being the contribution of the dicretised radiation modes. As the distance of reflecting boundaries from the device is increased, the eigenvalues of the discretised radiation modes will tend to a continuous distribution. In the limiting case, the radiation contribution in (15) becomes an integral. Hence:

$$(16)\qquad P_{loss} = \sum_{j=-J,j\neq 0}^{J}\left[\frac{K_{j1}}{M}\frac{\sin\dfrac{(\overline{\beta}_1-\overline{\beta}_j)L}{2}}{\sin\dfrac{(\overline{\beta}_1-\overline{\beta}_j)L}{2M}}\right]^2 + \int\limits_{\beta\leq-\beta_0,\beta\geq\beta_0} K_R^2(\beta)\left[\frac{\sin\dfrac{(\overline{\beta}_1-\beta)L}{2}}{M\sin\dfrac{(\overline{\beta}_1-\beta)L}{2M}}\right]^2 d\beta$$

where β_0 is the propagation constant of the first radiation mode and K_R the coupling distribution from the fundamental mode into the radiation modes (which will generally vary with the propagation constant β of the radiation mode).
Assuming that K_R is a continuous function of β, the integral expression for the radiation contribution in (16) will NOT exhibit the resonant peaks as L is varied, since the kernel

$$\frac{\sin\dfrac{(\overline{\beta}_1-\beta)L}{2}}{M\sin\dfrac{(\overline{\beta}_1-\beta)L}{2M}}$$

is a smooth, bounded function for all L.
The absence of these resonant lengths is physically explained by the fact that a negligible amount of light scattered into radiation modes from one taper step is likely to couple back into guided modes in neighbouring steps, so that no coherence effects as the one in figure 4 can occur.

7. Application.

We find that as the length of the *step taper* is increased from zero the power loss decreases as it would do for a *smooth taper* but then increases to a peak at the first resonant length of the highest guided mode – eqn. (13). The peak amplitude is of the same order as the coupling into this mode at a butt join. These peaks recur periodically as we increase the taper length further.
Therefore when designing *step tapers* it is desirable to keep their length below the first resonant length. The resonant length increases by augmenting the number of steps M in the taper. In practice these are limited by the resolution in the lithographic fabrication process: $M<\Delta W/\delta w_{min}$, where δw_{min} is the minimum step size allowed by the lithographic fabrication process and ΔW is the difference in widths of the guiding core at the taper ends. This puts a restriction on δL – the distance along the device between steps (δL=L/M). Eqn. (13) implies a safe upper limit for δL of

(17) $$\delta L_{max} = \frac{\pi}{\beta_0 - \beta_1}$$

- this being half the resonance length. Above this value, the taper efficiency will decrease again. In Figure 7 we plot some values for δL_{max} for a simple example of a 1D step index waveguide, at

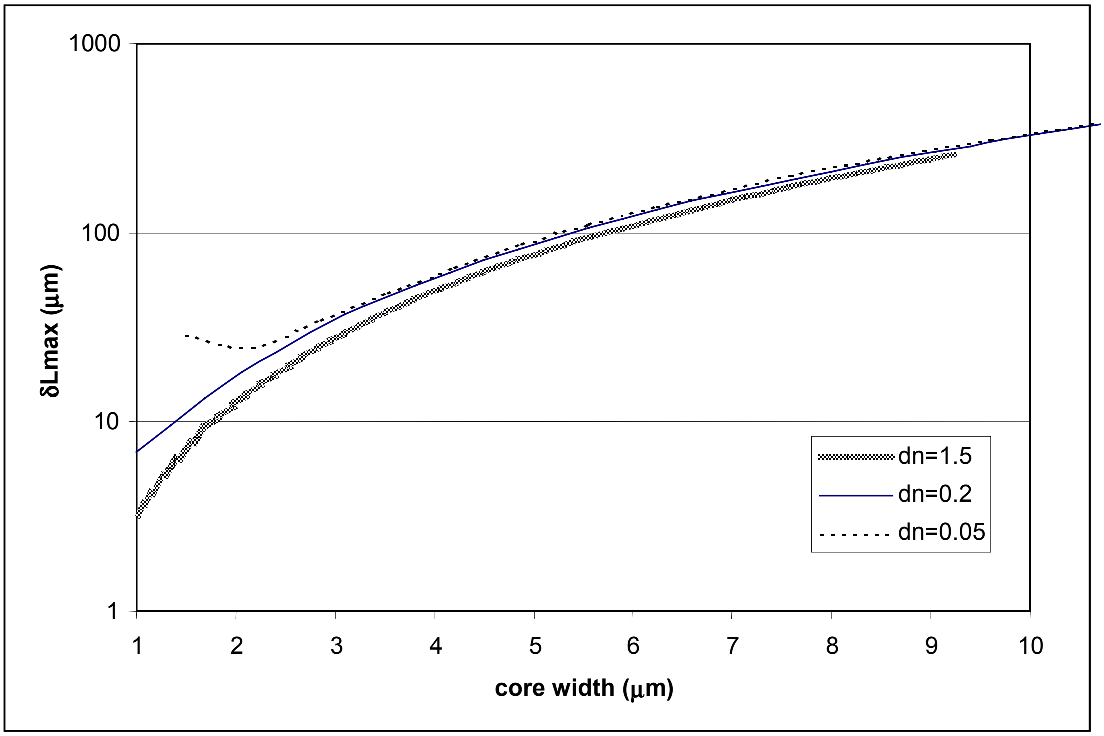

Figure 7 – variation of δLmax (the distance between steps) for a 1D step index waveguide.

a wavelength of 1.55um. The core refractive index in each case is 3.5 and the figure shows values of δL_{max} as a function of core width, for Δn values of 0.05, 0.2 and 1.5.

8. Conclusions

We have done an approximate **multimodal** analysis of the *step taper*. We have derived an expression (11) for the power lost from the excited input fundamental mode valid for large number of steps M. By setting M to infinity, (11) gives the power loss for a *smooth taper*, (14), which is a more general result than that found in various texts [3] derived using coupled mode theory. The results indicate the existence of resonant taper lengths at which the power coupled from the fundamental mode into another mode reaches a sudden peak. Worthy of a note is that at these lengths the power may couple into a backward instead of a forward travelling mode, so that significant reflections may appear in structures with a large enough refractive index contrast between the core and cladding.

These results contradict the conventional wisdom that "longer is better" for real tapers – making a taper too long may reduce its efficiency. We present simple formulae that can be used to determine how long the taper can be made without hitting the effect of the steps.

9. References.

1. I. Lealman, L. Rivers, M. Harlow, S. Perrin and M. Robertson, Electron Letter 30 (12) pp. 857-859, 1994.
2. F. Wijnands, C.G. Crookes, et al, Proc. IEEE LEOS, Paper FA1, pp. 283-284, 1998.
3. A.W. Snyder, J.D. Love *,"Optical Waveguide Theory"* , Chapman and Hall, 1991.